Differential Geometry of Singular Spaces and Reduction of Symmetries

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1 Differential geometry of singular spaces

A colloquium type lecture for general audience reviewing the theory of differential spaces and its applications.

1.1. Differential structures
1.2. The category of differential spaces
1.3. Manifolds and subcartesian spaces
1.4. Derivations and vector fields
1.5. Integration of vector fields and their families
1.6. Stratified spaces
1.7. Proper action of a Lie group on a manifold
1.8 Applications to reduction

2 Techniques of differential geometry

A technical lecture for students who want to use differential geometry to study singular spaces. It will consist of proofs or outlines of proofs of selected theorems

Differential geometry can be thought of as algebraic geometry in the smooth category. One can use all applicable techniques of algebraic geometry as well as integration.

1.1. Integration of derivations.

1.2. Generalized Stefan-Sussmann Theorem.

1.3. Outline of the proof that the space P/G of orbits of a proper action of a Lie group G on a manifold P is subcartesian and that the orbit type stratification of P/G is given by orbits of the family $\mathfrak{X}(P/G)$ of all vector fields on P/G.

3 Singular symplectic reduction

A non-technical lecture discussing applications of differential geometry to symplectic reduction.

Singular reduction in Hamiltonian mechanics describes the structure of the orbit space P/G of a proper action of a Lie group G of symmetries of a symplectic manifold (P, ω). Since we know that P/G is stratified, we need to describe the interplay between the stratification structure of P/G and the structure on P/G induced by the symplectic form ω.

1.1. Poisson structure of $C^\infty(P/G)$.

1.2. Orbit type stratification of P/G as the partition of P/G by the family $\mathfrak{X}(P/G)$ of all vector fields.

1.3. Symplectic singular foliation of strata of P/G by orbits of the family $\mathfrak{P}(P/G)$ of Poisson vector fields.

1.4. Level sets of the momentum map.

1.5. Reduction by stages.

4 Further examples of reduction

A non-technical lecture describing application of differential geometry to reduction of other systems.

1.1. Non-holonomic reduction.

1.2. Reduction of Poisson structures.