Short Course on Elliptic and Spectral Inverse Problems on Riemannian Manifolds

Matti Lassas, University of Helsinki

Fields Institute, Room 230

Abstract:

We consider the uniqueness results and counterexamples for inverse problems for the elliptic and hyperbolic equations on Riemannian manifolds. In applications, these problems arise in imaging of an anisotropic medium. The inverse problems in an anisotropic medium are not usually uniquely solvable as a change of coordinates changes the equation but does not change the boundary data. This point of view makes it possible to consider both uniqueness results and construct counterexamples.

In the uniqueness results the goal is to show that the boundary data corresponding to some measurements made on the boundary of the manifold determine uniquely the isometry type of the Riemannian manifold. For elliptic equations the boundary data corresponds to the Dirichlet-to-Neumann map on the boundary and for the spectral problems the boundary data can be for instance the eigenvalues and boundary values of the Neumann eigenfunctions of the Laplace-Beltrami operator. In applications, the reconstruction of the isometry type of the manifold means the imaging of an unknown medium in the travel-time coordinates.

To obtain counterexamples for uniqueness of inverse problems, we consider singular deformations of a domain. This leads to degenerate Riemannian metrics, or conductivity tensors that appear in boundary measurement similar to a homogeneous domain.
The counterexamples for solvability of inverse problems are closely related to the recent theoretical and experimental progress on making objects invisible to detection by electromagnetic waves, acoustic waves and quantum waves, that is, the so-called invisibility cloaking.

We will consider the following topics on the course:

-The formulation of inverse problems on manifolds and the invariance of measurements in deformations of the domain. Applications for the invisibility cloaking and transformation optics are considered.
-Inverse problems for a Laplace-Beltrami equation on a Riemannian surface.
-Inverse boundary spectral problems and inverse problems for hyperbolic equations. We review results on reconstruction of a Riemannian manifold and metric from the time domain data.