

The mod p representation theory of p -adic groups.

This course will give an introduction to the smooth representation theory of a p -adic reductive group (like $\mathrm{GL}_n(\mathbb{Q}_p)$) over $\overline{\mathbb{F}}_p$. The study was begun by Barthel–Livné for the group $\mathrm{GL}_2(F)$, where F is a finite extension of \mathbb{Q}_p . Breuil completed the classification of irreducible representations of $\mathrm{GL}_2(\mathbb{Q}_p)$ (having a central character), and related these representations to two-dimensional representations of $\mathrm{Gal}(\overline{\mathbb{Q}}_p/\mathbb{Q}_p)$ over $\overline{\mathbb{F}}_p$. This was one of the starting points for the investigation of the p -adic Langlands correspondence for $\mathrm{GL}_2(\mathbb{Q}_p)$. At this point it is not clear how the p -adic Langlands correspondence generalises to other groups. It is one of the goals of the course to explain some of the recent progress in the classification of irreducible representations for other groups, particularly for $\mathrm{GL}_n(F)$.

Some of the topics we will cover (time permitting):

- the work of Barthel–Livné for $\mathrm{GL}_2(F)$,
- irreducible $\overline{\mathbb{F}}_p$ -representations of $\mathrm{GL}_n(\mathbb{F}_p)$,
- parabolic induction and compact induction,
- unramified Hecke algebras and the Satake transform,
- generalised Steinberg representations,
- ordinary parts,
- classification of irreducible representations in terms of supercuspidal ones.

Prerequisites:

A good command of graduate algebra, including solid knowledge of the representation theory of finite groups over the complex numbers. Some familiarity with p -adic numbers.

Main references:

L. Barthel, R. Livné, *Irreducible modular representations of $\mathrm{GL}_2(F)$ of a local field*, Duke Math. J. 75, 1994, 261–292.

C. Breuil, *Sur quelques représentations modulaires et p -adiques de $\mathrm{GL}_2(\mathbb{Q}_p)$*

I, Compos. Math. 138, 2003, 165–188.

M. Emerton, *Ordinary parts of admissible representations of p -adic reductive groups I. Definition and first properties*, Astérisque 331 (2010), 335–381.

F. Herzig, *A Satake isomorphism in characteristic p* , Compos. Math. 147 (2011), no. 1, 263–283.

F. Herzig, *The classification of irreducible admissible mod p representations of a p -adic GL_n* , Invent. Math. 186 (2011), no. 2, 373–434.

Survey article for the p -adic Langlands programme:

C. Breuil, *The emerging p -adic Langlands programme*, Proceedings of I.C.M. 2010, Vol. II, 203–230.