

FREE PROBABILITY AND RANDOM MATRICES

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The goal of the course is to bring students to the frontier of research in free probability and its relation to random matrices in ten lectures. The lectures will present the basic facts of free probability and show how free probability can simplify and conceptualize the study of random matrices through a careful exploration of basic but instructive examples.

The course will consist of ten lectures and five exercise sets for those taking the course for credit. Most of the material will be from the recent book by Alexandru Nica and Roland Speicher: *Lectures on the Combinatorics of Free Probability*, Cambridge University Press, 2006, ISBN 0-521-85852-6. Preliminary versions of the book can be found at

<http://www.mast.queensu.ca/~speicher/papers/lectures-IHP.pdf> and <http://www.math.uwaterloo.ca/~anica/NOTES/notes.html>.

The class will meet for ten lectures starting on September 13 and ending on December 6; there will be no lectures on September 20, November 1, and November 15 to allow attendance at the program workshops.

**September 13, Lecture 1:** the distribution of a Gaussian random variable, moments and cumulants, multivariate moments and cumulants, Wick's formula for the moments of a multivariate Gaussian random variable, genus expansion for the moments of the GUE, asymptotic freeness of independent GUE's, free random variables.

Further topics include:

- free group factors, basic properties of freeness, and free cumulants.
- the  $R$ -transform and the  $S$ -transform.
- analytic methods, the Cauchy and Stieltjes transforms.
- compression of factors and fundamental groups of factors.
- free entropy.
- Haar unitaries, asymptotic freeness of random matrices, and the Weingarten function.
- operator-valued free probability

In addition to the book of Nica and Speicher mentioned above there are:

- P. Deift, *Orthogonal polynomials and random matrices : a Riemann-Hilbert approach*, Amer. Math. Soc., 2000,
- F. Hiai and D. Petz, *The Semicircle Law, Free Random Variables and Entropy*, Amer. Math. Soc. 2000,
- M. Mehta, *Random Matrices*, 3rd ed., Elsevier, 2004,
- D. Voiculescu, K. Dykema, A. Nica, *Free Random Variables*, Amer. Math. Soc. 1992.