ASHKAN AAZAMI
University of Waterloo

Approximating the Power Dominating Set problem

The power dominating set (PDS) problem is the following extension of the well-known dominating set (DS) problem: find a smallest-size set of nodes S that power dominates all the nodes, where a node v is power dominated if (1) v is in S or v has a neighbor in S, or (2) v has a neighbor w such that w and all of its neighbors except v are power dominated. A greedy algorithm for DS achieves logarithmic approximation guarantee. We show that different variations of the greedy algorithm perform poorly in PDS. The best known hardness threshold for PDS is logarithmic (this means that modulo the $P \neq NP$ conjecture, no poly time algorithm can find a solution whose cost is within a logarithmic factor of the optimal cost) which is proved by a reduction from DS problem. We also show an improved hardness threshold for PDS. This talk is based on a joint work with Michael Stilp. Coauthors: Michael Stilp (University of Waterloo)

JORDAN BELL
Carleton University

Cyclotomic $\mathcal{R}$-orthomorphisms and character sums

I will introduce $\mathcal{R}$-orthomorphisms of finite fields, and use them to give nontrivial bounds for certain character sums. I will give constructions for $\mathcal{R}$-orthomorphisms by cyclotomic mappings.

CHRISTINA BOUCHER
University of Waterloo

Graph Isomorphism Completeness for Subclasses of Perfect Graphs

The inability to directly classify the Graph isomorphism (GI) problem into either of the conventional complexity classes P or NP-complete led to the definition of the computational complexity class GI. A problem is said to be GI-complete if it is provably as hard as graph isomorphism; that is, there is a polynomial-time Turing reduction from the graph isomorphism problem. It is known that the GI problem is GI-complete even for some special classes including regular graphs, bipartite graphs, chordal graphs and split graphs. In this talk we prove that deciding isomorphism of double split graphs, the class of graphs exhibiting a 2-join, and the class of graphs exhibiting a balanced skew partition are GI-complete. These results are considered in the context of perfect graphs and demonstrate that each of the graph classes Chudnovsky et al. consider to prove the Strong Perfect Graph Theorem are GI-complete. Coauthors: David Loker

ANDREA BURGESS
University of Ottawa

Colouring even cycle systems

An m-cycle system of order n is a decomposition of the complete graph K_n into m-cycles. An m-cycle system is said to be weakly k-colourable if its vertices can be partitioned into k colour classes such that no cycle has all of its vertices the same colour. A cycle system's chromatic number is the smallest value of k for which the system is weakly k-colourable. While colourings of 3-cycle systems, or Steiner triple systems, have been widely studied, less is known regarding colourings of m-cycle systems in general. In this talk, we present some results on weak colourings of m-cycle systems for which the cycle length m is even, in particular, the result that for any integers $r \geq 2$ and $k \geq 2$, there is a k-chromatic $(2r)$-cycle system. We illustrate with examples of constructions of cycle systems with prescribed chromatic number. Coauthors: David Pike (Memorial University of Newfoundland)

EHSAN CHINIFOROOSHAN
University of Waterloo

Coloring Geometric Hypergraphs Induced by 3D Boxes

We consider the problem of coloring geometric hypergraphs induced by 3-dimensional boxes. Geometric hypergraphs, which were introduced by Smorodinsky, are hypergraphs $H = (V, \mathcal{E})$ where V is a set of regions and $\mathcal{E} = \{U \subset V \mid \exists p \forall v \in V : p \in v \text{ iff } v \in U\}$. Smorodinsky proved that the chromatic number of geometric hypergraphs induced by axis-parallel rectangles is in $O(\log\ n)$ and conjectured the chromatic number of geometric hypergraphs induced by d-dimensional boxes is in $O(\log^d n)$. We provide an upper bound of $O(\log^7 n)$ on the chromatic number of geometric hypergraphs induced by 3-dimensional boxes. Coauthors: Narges Simjour

MEGAN DEWAR
Carleton University

Universal Cycles for Triple Systems

Universal cycles are generalizations of de Bruijn cycles. Suppose we are given a family $\mathcal{F}_n$ of combinatorial objects of rank n with $m = |\mathcal{F}_n|$. Assume that each $F \in \mathcal{F}_n$ is specified by some sequence $[x_1, x_2, \ldots, x_n]$, where $x_i \in A$, for some fixed alphabet A. $U = (a_0, a_1, \ldots, a_{m-1})$ is a universal cycle (Ucycle) for $\mathcal{F}_n$ if $[a_{i+1}, \ldots, a_{i+n}]$, $0 \leq i < m$, runs through each element of $\mathcal{F}_n$ exactly once, where index addition is performed modulo m. Ucycles can be constructed for a variety of families of combinatorial structures including permutations, partitions and k-subsets of an n-set. In this talk we consider the existence of Ucycles for block designs. In particular, we show that Ucycles exist for every cyclic triple system with $v \geq 13$ and arbitrary λ. The proof method is constructive, therefore, similar techniques can be applied to BIBDs with $k \geq 3$ and to PBDs. Coauthors: Brett Stevens

LI DONG
Carleton University

Combinatorial Decomposable Structures with Restricted Pattern

We are interested in generalizations of the cycle index of decomposable objects: permutations, polynomials over finite fields, 2-regular graphs, random mappings and so on. By studying the generating functions of these objects we can learn how the behavior of a combination of components look like when the length of the object goes to infinity.

PAUL ELLIOTT-MAGWOOD
University of Ottawa

Constructing Optimal Solutions to the 2-edge-connected Spanning Subgraph Problem

Given a complete graph on n vertices and nonnegative costs assigned to the edges, the 2-edge-connected Spanning Subgraph Problem (2EC) is that of finding a minimum cost 2-edge-connected graph which spans all n vertices. Monma, Munson, and Pulleyblank showed that if the costs assigned to the edges of the complete graph are metric then there exists an optimal solution to the 2EC which is edge-minimally 2-edge-connected, is 2-vertex-connected, has maximum valency 3, and the removal of any pair of edges leaves a bridge in at least one of the remaining components. For this presentation, we will show how to construct all the graphs which have these properties using their ear decompositions, their necklaces, and their min-cut cactii. Coauthors: Sylvia Boyd

SHONDA GOSSELIN
University of Ottawa

Vertex-Transitive and Self-Complementary 3-Hypergraphs

A 3-hypergraph is a pair (V, E) in which V is a finite set or vertices and E is a set of 3-subsets of V called edges. An isomorphism between the 3-hypergraphs $X = (V, E)$ and $Y = (W, F)$ is a bijection from V to W which induces a bijection between E and F. If such a bijection exists, we say that X and Y are isomorphic. An automorphism of X is an isomorphism from X to X. The complement X^C of a 3-hypergraph $X = (V, E)$ is the 3-hypergraph $X^C = (V, E^C)$, in which E^C is the set of all 3-subsets of V which are not in E. A 3-hypergraph X is called self-complementary if it is isomorphic to its complement

X^C. A 3-hypergraph $X = (V, E)$ is vertex-transitive if for any two vertices v, w in V, there exists an automorphism of X which maps v to w.

In this talk, some results regarding the possible orders of vertex-transitive, self-complementary 3-hypergraphs will be discussed, and some constructions will be presented. In particular, we will investigate the structure of vertex-transitive and self-complementary 3-hypergraphs of prime order.

HAMED HATAMI
Department of Computer Science, University of Toronto

Fourier spectrum of Boolean functions.

In this talk I will discuss some results about the Fourier spectrum of the Boolean functions. Roughly speaking these results show that when the Fourier spectrum is concentrated on the first levels then the function can be approximated by a function which depends on a few number of variables.

CARLOS HOPPEN
University of Waterloo

On geodesics in random regular graphs

The application of probabilistic methods to combinatorics, introduced by P. Erdös in the late 1940's, has revealed itself a very fruitful approach to several questions in discrete mathematics. In the present talk, we shall illustrate the use of basic probabilistic tools through the study of geodesics in a random regular graph $G \in \mathcal{G}_{n,d}$, where a geodesic is a shortest path between two vertices in the graph and d is a constant. We prove that, with high probability, two vertices u, v lie at distance close to $\log_{d-1} n$ from each other. Moreover, we obtain the probability of two vertices u, v being connected by more than one geodesic, and we show that, with high probability, the middle points of two distinct geodesics between the same endpoints are also at distance close to $\log_{d-1} n$. Coauthors: Pawel Pralat

ROBERT JAMISON
Clemson University

On the Chromatic Spectrum of Decompositions of Graphs

The idea of a decomposition of graph occurs both in the study of intersection graphs and in combinatorial design theory. The former deals mostly with sparse graphs, the latter with complete and complete multipartite graphs. Here we will explore a theory for decompositions of general graphs which incorporates both these endeavours.

Let $\mathcal{K}$ be a family of graphs. A $\mathcal{K}$-*decomposition* $\mathcal{D}$ of a graph $H = (V, E)$ is a partition of the edge set E of H so that for each part P of the partition, the subgraph of H induced by P is isomorphic to a graph in $\mathcal{K}$. The graph H is called the *host graph* for the decomposition. The subgraphs of H induced by the parts of the partition are called the *blocks* of the partition, and the graphs in $\mathcal{K}$ are called the *block prototypes* for the decomposition.

The *intersection graph* $I(\mathcal{D})$ of the decomposition $\mathcal{D}$ has a vertex for each block of the partition and two blocks A and B are adjacent iff they share a common node. The *chromatic index* of a decomposition $\mathcal{D}$ is the chromatic number of $I(\mathcal{D})$. Thus the *chromatic index* $\chi'(\mathcal{D})$ of an $\mathcal{K}$-decomposition $\mathcal{D}$ of H is the minimum number of colors required to color the subgraphs in the decomposition so that subgraphs which share a node get different colors. The $\mathcal{K}$-*chromatic spectrum* $Spec_{\mathcal{K}}(H)$ is the set of all values of $\chi'(\mathcal{D})$ over all $\mathcal{K}$-decompositions $\mathcal{D}$ of H.

We will consider issues such as

(a) the computational complexity of determining the spectrum; (b) the uniqueness of the spectral value for trees; (c) lacunae in the sprectrum; (d) decompositions by matchings. Coauthors: Eric Mendelsohn

LAP CHI LAU
Department of Computer Science

On Steiner Rooted-Orientations of Graphs and Hypergraphs

Given an undirected hypergraph and a subset of vertices S with a specified root vertex r in S, the Steiner Rooted-Orientation problem is to find an orientation of all the hyperedges so that in the resulting directed hypergraph the "connectivity" from the root r to the vertices in S is maximized. This is motivated by a multicasting problem in undirected networks as well as a generalization of some fundamental problems in graph theory. Our main results are the following approximate min-max relations:

- Given an undirected hypergraph H, if S is 2k-hyperedge-connected in H, then H has a Steiner rooted k-hyperarc-connected orientation.
- Given an undirected graph G, if S is 2k-element-connected in G, then G has a Steiner rooted k-element-connected orientation.

Both are optimal in terms of the connectivity bounds. These also imply the first polynomial time constant factor approximation algorithms for both problems. Coauthors: Tams Kirly

J. BOWMAN LIGHT
Clemson University

On a Problem of Eric Mendelsohn on Edge-Intersections of Graphs

Given two graphs G and H sharing the same vertex set, the *edge-intersection spectrum* of G and H is the set of possible sizes of the intersection of the edge sets of both graphs. For example, the spectrum of two copies of the path P_n is $\{0, 1, ..., n-1\}$, and the spectrum of two copies of the star $K_{1,r}$ is $\{1, r\}$. The intersection spectrum was initially studied for designs by Lindner and Fu and was originally extended to graphs by Eric Mendelsohn. We will examine the spectra of several types of graphs, both when $G = H$ and when $G \neq H$, and show that any set of distinct positive integers always can be obtained as the intersection spectrum of two graphs. Coauthors: Robert E. Jamison (Clemson University)

JASON LOBB
Carleton University

A Gray Code of Mixed Radix N-Tuples of Constant Weight

A mixed radix Gray Code is a sequence of n-tuples $(x_n, x_{n-1}, \cdots, x_1)$ with $0 \leq x_i \leq s_i$ for $i = 0, 1, \cdots, n$. Let $C(k : x_n, x_{n-1}, \cdots, x_1)$ be the n-tuples such that $\sum_{i=1}^{n} x_i = k$. We prove that the order of occurance of the fixed weight n-tuples in the standard mixed radix reflected Gray code ordering of all n-tuples is also a minimal change ordering of $C(k : x_n, x_{n-1}, \cdots, x_1)$. The minimal change is such that the successive n-tuples only differ in two positions and these positions differ in value from the predecessor by 1. We present an algorithm to calculate the successor in this minimal change ordering. Coauthors: Brett Stevens

DAVID LOKER
University of Waterloo

Combinatorial Auctions and Winner Determination on Graphs

Combinatorial auctions provide a useful mechanism for representing auctions on multiple goods where a bidder's valuation on some bundles of items may not equal the sum of the bidder's value for each individual item in those bundles. The winner determination problem is the problem of allocating goods to bidders such that the seller's revenue is maximized, and this problem is known to be NP-hard. We look at a specific representation for bidder's valuation functions and from this representation we formulate combinatorial

auctions as a graph and show how the winner determination problem can be solved using weighted independent set. Coauthors: Christina Boucher

ARIANE MASUDA
School of Mathematics and Statistics - Carleton University

The number of permutation binomials over $\mathbb{F}_{4p+1}$ where p and $4p+1$ are primes

We give a characterization of permutation polynomials over a finite field based on their coefficients, similar to Hermite's Criterion. Then, we use this result to obtain a formula for the total number of monic permutation binomials of degree less than $4p$ over $\mathbb{F}_{4p+1}$, where p and $4p+1$ are primes, in terms of the numbers of three special types of permutation binomials. Coauthors: Daniel Panario (Carleton University), Qiang Wang (Carleton University)

SHENGJUN PAN
Department of Combinatorics & Optimization, University of Waterloo, ON, N2L 3V5

The Crossing Number of K_{11} is 100

Guy conjectured that the crossing number $cr\ (K_n)$ of the complete graph K_n is $Z(n) = \frac{1}{4}\lfloor\frac{n}{2}\rfloor\lfloor\frac{n-1}{2}\rfloor\lfloor\frac{n-2}{2}\rfloor\lfloor\frac{n-3}{2}\rfloor$. He proved this for $n \leq 10$ and also determined that, for $n = 4, 5, 6, 7, 8$, the number of optimal drawings of K_n is $1, 1, 1, 5, 3$, respectively. In this talk we use some simple counting properties to provide the basis of an algorithm which we programmed to show that $cr\ (K_{11}) = Z(11)$. In particular, we determine that K_9 and K_{10} have 3080 and 5679 optimal drawings, respectively. Along the way, we answer affirmatively an open question of Brodsky *et al.* by showing that every good drawing of K_n induces a 3-connected planar graph. Coauthors: Bruce Richter (University of Waterloo)

EUN-YOUNG CHRISTINA PARK
Department of Electrical and Computer Engineering, University of Toronto

Repeated Steiner Systems

A Steiner System is a collection of k-subsets (called blocks) of a v-set such that every t-subset belongs to exactly one block. While blocks in Steiner Systems consist of distinct elements, we consider a collection of k-multisets (hence, allowing repetition of elements in a block) of a v-set such that every t-multiset belongs to exactly one block. We call this system a Repeated Steiner System and denote it as RS(t, k, v). Although the problem appears to be identical to multiset designs, the number of t-multisets in each block is counted differe ently from that in multiset designs, which changes the problem significantly. An RS(t, k, v) provides a natural solution to a variation of a multiset batch code problem, which aims to amortize computational complexity in Private Information Retrieval (PIR). Our way of counting is justified from this perspective to reduce storage requirement. We study RS(t, t + 1, v) and show constructions for RS(2, 3, v) for all positive integers, v, and RS(3, 4, v) for infinitely many values of v. Coauthors: Ian F. Blake (University of Toronto)

BRETT STEVENS
School of Mathematics and Statistics, Carleton University

Covering Arrays: An Exploration of Diverse Viewpoints on an Applied Problem.

Covering arrays are combinatorial objects that have several applications, including software and hardware testing. We will examine covering arrays from different points of view including design theory, extremal set theory, information theory and probability. We will also introduce several recent generalizations that bring in new points of view notably various aspects of graph theory.

MARC TEDDER
University of Toronto

Dynamically recognizing distance-hereditary graphs under edge addition and deletion

A dynamic graph algorithm is interested in maintaining information about a graph or verifying some property of a graph as it undergoes changes in the form of vertex and edge additions and deletions; an example of information to be maintained is the distance function, and an example of a property to be verified is membership in a class of graphs. In this talk I will describe an optimal dynamic algorithm that verifies membership in the class of distance-hereditary graphs under a sequence of edge additions and deletions. A graph is distance-hereditary if and only if every induced path between all pairs of vertices has the same length.

GABRIEL VERRET
University of Ottawa

Shifts in Cayley Graphs

An automorphism of a simple graph is called a shift if it maps every vertex to an adjacent one. We consider which Cayley Graphs have shifts. We also consider for which groups do all Cayley Graphs on these groups admit a shift.

JACQUES VERSTRAETE
University of Waterloo

Cycles of prescribed lengths in graphs

The central theme of this talk is to study the largest possible average degree $d(n, S)$ of an n-vertex graph with no cycle of length from a given set of positive integers S. When S contains only odd numbers, the extremal graphs are complete bipartite graphs, so $d(2n, S) = n$ in this case. When S contains even numbers, the problem becomes notoriously difficult. The case $S = \{2k\}$ is Erdos' Even Cycle Theorem.

In this talk I will give a short proof of this theorem, which states $d(n, S)$ is at most about $n^{1/k}$. The method used to prove this allows us to consider any set S of forbidden even cycle lengths: a general theorem will be presented which shows that apart from chaotic looking sets S, $d(n, S)$ is at most about $\exp(\log *n)$. This is motivated by a one thousand dollar conjecture of Erdos that $d(n, S)$ is a constant when S is the set of powers of two. Very surprisingly, our result is tight: there exist sets S for which $d(n, S)$ is roughly $\exp(\log *n)$.

LATIFA ZEKAOUI
University of Ottawa

Mixed Covering Arrays on Graphs

Covering arrays are generalizations of orthogonal arrays that are used for testing software, networks and circuits. Let N, g, k be positive integers. A covering array is a k by N array with entries from Z_g such that any 2 by N subarray contains every ordered pair of elements from Z_g in some of its columns. In this talk, we look at a further generalization of covering arrays that considers both mixed alphabet sizes for different rows and a graph structure on the rows that prescribes the pair of rows for which we require the covering property. A (standard) covering array is a particular case where all alphabets are the same and G is the complete graph. We give optimal constructions of mixed covering arrays on graphs for specific classes of graphs. Coauthors: Karen Meagher and Lucia Moura